On the cosmological constant and vacuum energy

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(April, 2000)

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We present our view on the problem of the cosmological constant and vacuum energy. It is pointed out that only relevant modes of the vacuum fluctuation, whose wavelengths are conditioned by the size, homogeneity, geometry and topology of the present Universe, contribute to the cosmological constant. As a result, the cosmological constant is expressed in terms of the size of the Universe and the three

fundamental constants: the velocity of light, Planck and Newton gravitational constants. Its present value remarkably agrees with the recent observations and its dependence on the size of the Universe confronts with observations.

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Introduction. The recent claims on the observational indications of the acceleration of the expansion of the Universe [1] indicate that the Ω_Λ attributed to the vacuum-energy density is of the same order of magnitude as the present matter density of the Universe. Though the accuracy of observational data is still a subject for further analysis and more data are required for the definite conclusions of the deviation from the standard Hubble expansion, these observations have greatly revived the interest in the long-standing problem of the cosmological constant for not only its small value, but also its closeness to the critical density of the Universe.

The cosmological term (Λ-term) $\Lambda g_{\mu\nu}$ was introduced by Einstein to incorporate the General Relativity with the Mach principle. To interpret apparently observed behavior of redshift distribution of quasars within cosmological models of the Λ-term, Zeldovich [2] revealed the origin of the Λ-term being attributed to the zero-point energy of the vacuum: the vacuum energy-momentum tensor $\tilde{T}_{ik} = \Lambda g_{ik}$ and the negative vacuum pressure $\tilde{p}$ relating to the vacuum-energy density $\tilde{T}_{00} = \tilde{\rho} = -\tilde{p}$.

In the context of local quantum field theories in a flat space, the vacuum fluctuations of various quantum fields result in the non-vanishing vacuum-energy density given by (up to an irrelevant constant),

$$\tilde{\rho} = \frac{1}{2V}\sum_{\mathbf{k}} d^B_{\mathbf{k}}\epsilon_B(|\mathbf{k}|) - \frac{1}{V}\sum_{\mathbf{k}} d^F_{\mathbf{k}}\epsilon_F(|\mathbf{k}|), \tag{1}$$

where $\epsilon_{B,F}(|\mathbf{k}|)$ is the possible spectrum of all quantum bosonic and fermionic fields, V is the volume of the 3-dimensional space and the summation is performed over all possible $\mathbf{k}$ states with degeneracy $d^B_{\mathbf{k}}$ and $d^F_{\mathbf{k}}$. It cannot be precluded the contribution of the quantum gravity in eq.(1).

In the four dimensional flat space-time R^4 and the continuous spectrum $\epsilon_{B,F}(|\mathbf{k}|)$ of free quantum fields with a ultraviolet cutoff at the Planck scale $\Lambda_p \simeq 10^{19}$GeV, the vacuum-energy density (1) is of the order of $\Lambda_p^4 \simeq 10^{76}$GeV. This is 10^{123} times larger than the present observation, 10^{-47}GeV, and it is unclear how they incorporate with each other unless an extremal fine-tuning is made. This is the cosmological constant problem, as discussed in details by Weinberg in [3], which challenges the fundamental theories, both the local quantum field theory and General Relativity.

The theoretical approaches to this bizarre problem can be briefly classified into three categories: (i) fundamental physics, (ii) "quintessence" and (iii) the anthropic principle. In the first category, all negative energy states are fully filled and the mean-value of the vacuum-energy density is positive, the small Λ-term is related to: the gravitational potential between virtual particles; various scales of fundamental physics, such as electroweak processes, inflationary particle creations, et al. [4]. The elegant supersymmetry forces in eq.(1) bosonic and fermionic contributions to precisely cancel each other and its breaking scale gives rise to a Λ-term. In the second category, "quintessence" [5,6] postulates a new self-interacting scalar field Φ with a potential $V(\Phi)$, incorporating within inflationary model and quantum cosmology [7]. In the third category, the anthropic principle describes the probability of the Λ-term that we live with, conditioned by the necessity for the suitable evolution of intelligent life [8]. These studies of the physical background of the possible non-zero and small Λ-term are of definite interest and very fruitful for making steps toward understanding the cosmological constant problem. Nevertheless, it seems still far from achieving a conclusive and natural solution, which has a self-consistent connection to the observed value of the cosmological constant.

In this letter, we present an alternative view on the problem of the cosmological constant and vacuum energy. Namely, at the present epoch the cosmological constant is arised only from the contributions from *the relevant vacuum fluctuations* whose wavelengths are conditioned by the *size,*

homogeneity, geometry and *topology* of the Universe. Adopting the flat Friedmann-Robertson-Walker (FRW) Universe with the topology $T\otimes R^3$ as an example, we demonstrate this idea and obtain that the cosmological constant Λ-term depends on the radius $a(t)$ of the Universe, as $a^{-4}(t)$, and its present value is close to observational constraints.

The Λ-term and relevant vacuum fluctuation. A very large amount of the vacuum-energy density (1), by the order of magnitude, does not strongly depend on the details of spectra $\epsilon_B(\mathbf{k})$ and $\epsilon_F(\mathbf{k})$, i.e., massive or massless, free or interacting one. Instead, it strongly depends on the number of high energy modes, since the vacuum-energy density (1) is mainly contributed from the high energy modes of vacuum fluctuations of various quantum fields at short distances.

In the general relativity, the gravity must be generated by all kinds of energy-mass, including the vacuum energy created by vacuum fluctuations of various quantum fields. However, it seems very mysterious why such a hugh vacuum-energy density $O(10^{76})$GeV

(1), contributed from each spice of quantum fields, is absent in the right-handed side of the Einstein equation, thus has no any effect on the classical gravity. It is conceivable that this fact could be due to two possibilities at the distance of the Planck length L_{pl}: (i) hidden symmetries forcing the vacuum-energy density (1) to be exactly zero; (ii) a certain type of dynamical cancelations between quantum gravity's contributions and quantum field's contributions in the vacuum-energy density (1) [9]. We are not ambitious to cope with this problem here. Instead, we assume that the mean-value of the vacuum-energy density (1) is zero and does not contribute to the right-handed side of the Einstein equation.

On the other hand, it is equally unclear how the left-handed side of the Einstein equation describing the geometry and the large-scale Universe at present epoch can contain a non-vanishing cosmological term $\Lambda g_{\mu\nu}$. This is the problem we would like to address in this paper. We first define the notion of *the vacuum fluctuation*: it is a causally-correlated fluctuation of the vacuum, upon the zero mean-value of the vacuum-energy density (1). This has to be distinguished from the fluctuations of various quantum fields in the vacuum, which contribute to eq.(1). The non-vanishing cosmological term $\Lambda g_{\mu\nu}$ is originated from the relevant modes of the vacuum fluctuation. The precise definition of the relevant modes of the vacuum fluctuation will be given in due course.

We introduce a complex scalar field ϕ to effectively mimic the causally-correlated vacuum fluctuations upon the zero mean-value of the vacuum-energy density (1). The simplest coordinate-invariant action $\tilde{S}$ for the quantum scalar field $\tilde{S}$ is given by ($\hbar = c = 1$)

$$\tilde{S} = \frac{1}{2}\int d^4x\sqrt{-g}\Big[g^{\mu\nu}\phi_{,\mu}\phi^*_{,\nu} + (m^2 + \xi\mathcal{R})\phi\phi^*\Big], \tag{2}$$

where m is an effective mass of the scalar field and ξ is the coupling constant to the Riemann scalar $\mathcal{R}$. In terms of the Riemann tensor $\mathcal{R}_{\mu\nu}$ and the energy-momentum tensor $T_{\mu\nu}$ of the classical matter, the Einstein equation is written as

$$\mathcal{R}_{\mu\nu} - \frac{1}{2}\mathcal{R}g_{\mu\nu} + 8\pi G\langle\tilde{T}_{\mu\nu}\rangle_r = -8\pi G T_{\mu\nu}. \tag{3}$$

The cosmological term is described by an averaged energy-momentum tensor

$\langle \tilde{T}_{\mu\nu} \rangle_r$ of the quantum scalar field ϕ:

$$\langle \tilde{T}_{\mu\nu}(x) \rangle_r = -\frac{2}{\sqrt{-g}} \frac{\delta \ell n Z_r}{\delta g^{\mu\nu}(x)}, \quad Z_r = \langle 0|0 \rangle_r = \int [\mathcal{D}\phi \mathcal{D}\phi^*]_r \exp(-\tilde{S}), \tag{4}$$

which is averaged over the relevant modes of the vacuum fluctuation with the amplitude (the partition function Z_r) of transition between relevant modes of the vacuum fluctuation in the background of Einstein equation's solution $g_{\mu\nu}(x)$ and the global topology of the Universe.

Given the action $\tilde{S}$ (2) of the quantum scalar field ϕ in the curved space-time of the Universe with its topology, we can in principle determine a unique complete and orthogonal basis of wave-functions $u^r_{\mathbf{k}}(x)$ of relevant modes $\mathbf{k}$ of the scalar field:

$$\phi(x) = \sum_{\mathbf{k}} \Big(a_{\mathbf{k}} u^r_{\mathbf{k}}(x) + a^\dagger_{\mathbf{k}} u^{r*}_{\mathbf{k}}(x) \Big), \tag{5}$$

where $a_{\mathbf{k}}$ is the amplitude of the $\mathbf{k}$-th relevant mode. On this relevant basis, the partition function Z_r can be computed as

$$Z_r = [\det(M^r)]^{-1}, \quad M^r_{\mathbf{k},\mathbf{k}'} = \int d^4x \sqrt{-g} u^r_{\mathbf{k}}(x) (\Delta_x + m^2 + \xi R) u^{r*}_{\mathbf{k}'}(x) \tag{6}$$

$$\Delta_x = \frac{1}{\sqrt{-g}} \partial_\mu [\sqrt{-g} g^{\mu\nu} \partial_\nu]. \tag{7}$$

Diagonizing the hermitian matrix M^r, we obtain

$$\ell n Z_r = -\int d^4x \sqrt{-g} \int \frac{d^4k}{(2\pi)^4} \ell n(\lambda^r_{\mathbf{k}}), \tag{8}$$

where $\lambda^r_{\mathbf{k}}$ denotes the $\mathbf{k}$-th eigen-value of the matrix M^r. Thus, the averaged energy-momentum tensor $\langle \tilde{T}_{\mu\nu} \rangle_r$ (4) is given by,

$$\langle \tilde{T}_{\mu\nu} \rangle_r = g_{\mu\nu}(x) \int \frac{d^4k}{(2\pi)^4} \ell n(\lambda^r_{\mathbf{k}}), \tag{9}$$

where we approximately neglect the functional variation $\delta g_{\mu\nu}(x)$ of eigen-values $\lambda^r_{\mathbf{k}}$ in the logarithmical function. We identify the cosmological constant,

$$\Lambda = 8\pi G \int \frac{d^4k}{(2\pi)^4} \ell n \lambda^r_{\mathbf{k}}. \tag{10}$$

This clearly indicates that the cosmological constant is determined by the eigen-values of relevant modes of the scalar field in the background of the Einstein equation.

In eq.(2), the mass of the scalar field m is scaled by the masses m_f of virtual fermions and anti-fermions that are annihilated and created in vacuum fluctuations of quantum fields. Obviously, $m \ll \Lambda_p$, otherwise there would not be any vacuum fluctuations. Analogous to eq.(1), major vacuum-fluctuation modes contributing to the averaged energy-momentum tensor (9) is stemming from the high-energy range (m, Λ_p), and we can approximately neglect the mass term $m\phi^2$ in computations. Analogously, we approximately neglect the local interacting term $\xi\phi^2 R$, since the scale of the Riemann scalar $\mathcal{R}$ describing the large structure of the Universe is even very much smaller than m. One should not expect any significant coupling between very rapid variations of high-energy modes at short distances and the Riemann scalar $\mathcal{R}$ at long distances.

The Λ-term in the flat FRW Universe. We consider the FRW Universe described by a scale factor $a(t)$, curvatures $K = -1, 0, 1$ and interval

$$\begin{aligned} d^2s &= d^2t - a^2(t)\Big[(1 - Kr^2)^{-1}d^2r + r^2(d^2\theta + \sin^2\theta d^2\phi)\Big] \\ &= a^2(t)\Big[d^2\eta - \sum_{ij} h_{ij}dx^i dx^j\Big], \end{aligned} \tag{11}$$

where $d\eta = a^{-1}(t)dt$ is the conformal time and h_{ij} is the FRW 3-metric. Given the action (2) with $m = 0$ and $\xi = 0$, the massless scalar field ϕ obeys the equation of motion,

$$M^r\phi = \Delta_x u^r_{\mathbf{k}}(x) = 0, \quad u^r_{\mathbf{k}}(x) = \mathcal{Y}^r_{\mathbf{k}}(r,\theta,\phi)\chi^r_{\mathbf{k}}(\eta). \tag{12}$$

The function $\mathcal{Y}^r_{\mathbf{k}}(r,\theta,\phi)$ fulfills the equation:

$$\Delta^3\mathcal{Y}^r_{\mathbf{k}}(r,\theta,\phi) = -|\mathbf{k}|^2\mathcal{Y}^r_{\mathbf{k}}(r,\theta,\phi), \quad \Delta^3 = \frac{1}{\sqrt{-h}}\partial_i[\sqrt{-h}h^{ij}\partial_j]. \tag{13}$$

The function $\chi^r_{\mathbf{k}}(\eta)$ obeys the equation

$$\frac{\partial^2\chi^r_{\mathbf{k}}(\eta)}{\partial\eta^2} + |\mathbf{k}|^2\chi^r_{\mathbf{k}}(\eta) = 0, \quad \chi^r_{\mathbf{k}}(\eta) \sim e^{i\epsilon(|\mathbf{k}|)\eta} \tag{14}$$

where $\epsilon(|\mathbf{k}|) = |\mathbf{k}|$. The solution $\chi^r_{\mathbf{k}}(\eta) \sim e^{i\epsilon(|\mathbf{k}|)\eta}$ shows the positive spectrum $\epsilon(|\mathbf{k}|)$ of the massless quantum scalar field with respect to the Killing vector ∂_η for $\eta, t \to \infty$.

In order to find the relevant modes contributing to the cosmological constant Λ (10), we have to solve the eigenvalue equation (13) for a given size, homogeneity, geometry and topology of the Universe. We will consider for simplicity the flat FRW Universe $K = 0$, with the radius $a(t)$ and topology $T \times R^3$ where T is the time and R^3 is the compactified spatial manifold described by the coordinates (r, θ, ϕ). The general solution of Eq.(13) can be written as

$$\mathcal{Y}^r_{\mathbf{k}}(r, \theta, \phi) \sim j_l(k_r r) Y_{lm}(\theta, \phi), \tag{15}$$

where $Y_{lm}(\theta, \phi)$ is the spherical harmonic function and $j_l(k_r r)$ is the spherical Bessel function with the radial momentum k_r and the angular quantum number $l = 0, 1, 2....$ The eigen-value $\lambda^r_{\mathbf{k}}$ in eq.(8) is then given by,

$$(\lambda^r_{\mathbf{k}})^2 = k_t^2 + k_r^2 + \frac{l(l+1)}{r^2}, \tag{16}$$

where k_t is the temporal component of eigen-value $\lambda^r_{\mathbf{k}}$. Integrating over k_t in eq.(10) leads to

$$\Lambda = 8\pi G \int \frac{d^3k}{(2\pi)^3} \epsilon(|\mathbf{k}|), \quad \epsilon(|\mathbf{k}|) = \sqrt{k_r^2 + \frac{l(l+1)}{r^2}}, \tag{17}$$

up to an irrelevant integral constant independent of $|\mathbf{k}|$.

It is very crucial that in our problem, due to the cosmological principle, the angular quantum number l cannot be any other values except $l = 0$ in the general solution of eq.(15). In the other words, due to the homogeneity and isotropy of the Universe, there is not a chosen point $r = 0$ of the space-time where $\langle \tilde{T}(x) \rangle_r$ and $\phi(x)$ vanish, which just requires $l = 0$. Angular quantum numbers of relevant modes of the vacuum fluctuation ϕ must be zero. In addition, the quantum scalar field ϕ is confined within the manifold of the topology R^3 and we have a simple boundary value problem with the Dirichlet condition,

$$j_\circ(\alpha^n_\circ) = 0, \quad k^n_r = \frac{\alpha^n_\circ}{a(t)}, \tag{18}$$

where $\alpha^n_\circ$ is the n-th zero-point of the spherical Bessel function $j_\circ(x)$. The k^n_r denotes the radial momentum of relevant modes contributing the cosmological constant Λ. In our Universe $a(t) \gg 1$ and the asymptotic behavior of $j_\circ(x)$ is

$$j_\circ(k_r a(t)) \simeq \frac{2}{a(t)} \sin(k_r a(t)), \tag{19}$$

and we find $\alpha_\circ^n = k_r^n a(t) = n\pi, n = 0, \pm 1, \pm 2, \cdots$. The positive spectrum $\epsilon(|\mathbf{k}|)$ of relevant modes of the vacuum fluctuation is given by,

$$\epsilon(k_r) = k_r = \frac{\pi n}{a(t)}, \quad n = 0, 1, 2, \cdots. \tag{20}$$

As a result, we obtain the cosmological constant Λ,

$$\Lambda = 8\pi G \sum_l \frac{(2l+1)}{a^2(t)} \int \frac{dk_r}{(2\pi)} \sqrt{k_r^2 + \frac{l(l+1)}{a^2(t)}} = 8\pi G \frac{\pi}{a^4(t)} \sum_n n. \tag{21}$$

The Ω_Λ is then given by,

$$\Omega_\Lambda \equiv \frac{\Lambda}{8\pi G} = \frac{1}{2} \frac{\hbar c \pi}{2a^4(t)} N_{\max}(N_{\max} + 1), \tag{22}$$

where $N_{\max}$ is the maximum number of relevant modes in the radial direction. The "cosmological constant" Λ-term is inverse proportional to $a^4(t)$, appearing strong dependence on the radius $a(t)$ of the Universe, hence on the time elapsed from the Big Bang. Let us estimate the present value of the Ω_Λ. The maximum number of relevant modes in the radial direction is approximately given by

$$N_{\max} \simeq \frac{a(t)}{L_{pl}} \simeq 10^{61}, \tag{23}$$

where the present size of the Universe $a(t) \simeq 1 \cdot 10^{28}\, cm$ and the Planck length is $L_{pl} \simeq 1.6\, 10^{-33}\, cm$. This numerically yields the present value of the Ω_Λ

$$\Omega_\Lambda = \frac{\hbar c \pi}{2a^4(t)} N_{\max}^2 \simeq 5.5\, 10^{-28}\, g\, \mathrm{cm}^{-3}, \tag{24}$$

consistently with recent observations. The corresponding vacuum pressure is negative and thus accelerates expansion of the Universe.

Geometry and topology dependence. On the basis of dimensional analysis, eq.(10) indicates that Ω_Λ must be inverse proportional to $a^4(t)$ (22). The finite proportional coefficient in the result (22) certainly depends on both geometry and topology of the Universe. It is of definite interest to compute the cosmological term (10) in various backgrounds of geometry and topology of the Universe, although computations must be much more complicate. The main difficulty is in the formulation of the eigenvalue problem in manifolds of different topologies. Namely, in certain topologies the spectrum can be continuous, while in others even the problem itself cannot be formulated (Sobolev problem). Since we do not know the topology of the Universe *a priori*, any computation involving the eigenvalues cannot be unambiguous. In our problem we however expect that for other topologies of our interest, i.e. maximally symmetric manifolds, the result (22) can be changed within some small numerical factor, without any qualitative change. An alternative tool avoiding the difficulties of the eigenvalue problem in hyperbolic manifolds is the theory of dynamical systems which was used for the study of certain properties of CMB [10].

Indeed, the probability of creation of the Universe within the framework of quantum cosmology depends not only on the matter fields but also on the cosmological constant and the topology. Particularly, the S^3 topology has the highest probability determined via the wave function of the Universe, among the considered $R \times S^3$, (K=1), $R \times H^3/\Gamma$ (K=-1) R^4 (K=0) topologies [11] and $T \times S^3$, (K=0) [12]. In the case of inflationary Universe, the probabilities for the positively and negatively curved and flat geometries become comparable.

Conclusion and remarks. We conclude that the cosmological term in the left-handed side of the Einstein equation describing the present Universe is attributed to the vacuum fluctuation ϕ whose wavelengths are conditioned by the size, homogeneity, geometry and topology of the Universe. Due to the small anisotropy of the Universe, the angular quantum numbers l of the relevant modes of the vacuum fluctuations (21) could be finite numbers. As a consequence, the cosmological constant would be θ-dependent, which must not be more than the order of the anisotropy of the Cosmic Background radiation, 10^{-5}. We would like to stress several points as follows before ending this paper.

Observers within the background of the vacuum would never measure any physical effects of an absolute value of the vacuum-energy density (1). However, in certain circumstances, observers can possibly measure the energetic difference due to the vacuum fluctuations of various quantum fields interacting with an external and macroscopic source, which alters the spectrum of quantum field fluctuations. This source can be either a boundary or external potential. The Casimir effect is an example for the bosonic vacuum energy and the effect recently proposed [13] could be another for the fermionic vacuum energy.

The vacuum fluctuation ϕ interacting with the classical gravity appears as a cosmological term in the left-handed side of the Einstein equation describing the Universe at the present time. In the other words, the interaction between the vacuum fluctuation ϕ and classical gravity indicates that the vacuum fluctuation ϕ is entirely embedded into the Universe with its own size, geometry and topology. This means that such an interaction is non-local and extends over the whole Universe, which acts as a complex manifold restricting the vacuum fluctuation. To obtain the relevant value of the averaged energy-momentum tensor $\langle\tilde{T}_{\mu\nu}\rangle_r$ of the vacuum fluctuation as the cosmological term entering the Einstein equation for the Universe, the spectrum of the vacuum fluctuation should be the one determined by the Universe, rather than the one determined only by local quantum field theories. The cosmological principle based on the homogeneity and isotropy of the Universe plays an important role in determining the relevant spectrum of the vacuum fluctuation for the cosmological term.

It is worthwhile to mention that the cosmological term (22) is in fact an *absolute* value of the energy density of the vacuum fluctuation (9) in the Universe where we live in. If it was the energetic difference of the energy density (9) with and without presence of the classical gravity (Universe), the resulted cosmological term would be $\sim \hbar c/a^4(t)$, which can be obtained by analogous computations leading to the Casimir effect. This result is 10^{122} times smaller than the present observation of the cosmological constant.

Our scenario and result for the cosmological term confront with observations. Armed with the cosmological term (22), we have studied the acceleration of the Universe and acoustic peaks of the CMB and analysis will be presented in a separate paper [14]. If the determination of the cosmological constant and the acceleration of the Universe via the distant supernovae data would be confirmed, then the cosmological term that we obtained, should be a candidate responsible for the observed value of the cosmological constant and quantitatively explain the acceleration of the Universe. Beside, it is *a priori* not excluded that this complex scalar field describing the vacuum fluctuations could have some effects also on the large scale distribution of the matter.

There must be a certain connection between the cosmological term attributed to the vacuum fluctuation ϕ in the left-handed side of the Einstein equation and the "missing" energy-momentum tensor of the vacuum fluctuations of various quantum fields in the right-handed side of the Einstein equation. One is related to the vacuum fluctuation ϕ coupling to the gravitational field of long wave-lengths, as discussed in this paper. While another should be related to the vacuum fluctuations of various quantum fields coupling to the quantum gravitational field of short wave-lengths, which are important for the primordial epoch. We are not able to offer any further understanding and the question is completely open.

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